Nonlocal School on Fractional Equations
NSFE 2017
School Booklet

Department of Mathematics
Iowa State University

August 17–19, 2017

Courses

Regularity theory for fractional diffusion

Luis A. Caffarelli
The University of Texas at Austin

We will discuss some of the main ideas of fractional diffusion: quasilinear and fully non linear equations, and some phase transition problems, and how they connect to the second order theory.

Numerical methods for fractional diffusion

Ricardo H. Nochetto
University of Maryland, College Park

We present and analyze three finite element methods (FEMs) for the numerical approximation of fractional diffusion in bounded domains in any dimensions. The first FEM deals with the fractional spectral Laplacian and hinges on the extension to an infinite cylinder in one more dimension. The second FEM concerns the integral formulation of fractional Laplacian in the entire space. The third FEM is a Dunford-Taylor approach which applies to both definitions. We discuss rather delicate numerical issues that arise in the construction of reliable FEMs and in the a priori and a posteriori error analyses of such FEMs for both steady and evolution fractional diffusion, show illustrative simulations, and mention challenging open questions.

Conferences

An optimization-based coupling strategy for classical and nonlocal elasticity

Marta D'Elia

Sandia National Laboratories

The use of nonlocal models in science and engineering applications has been steadily increasing over the past decade. The ability of nonlocal theories to accurately capture effects that are difficult or impossible to represent by Partial Differential Equations (PDEs) motivates and drives the interest in this type of simulations. However, the improved accuracy of nonlocal models comes at the price of a significant increase in computational costs. As a result, it is important to develop local-to-nonlocal coupling strategies, which aim to combine the accuracy of nonlocal models with the computational efficiency of PDEs. We develop an optimization-based method for the coupling of nonlocal and local problems in the context of nonlocal elasticity. The approach formulates the coupling as a control problem where the states are the solutions of the nonlocal and local equations, the objective is to minimize their mismatch on the overlap of the nonlocal and local domains, and the controls are virtual volume constraints and boundary conditions. Numerical results for nonlocal diffusion and nonlocal elasticity in three-dimensions illustrate key properties of the optimization-based coupling method; these numerical tests provide the groundwork for the development of efficient and effective engineering analysis tools.

Hölder and Schauder estimates. Pointwise and semigroup strategies.

Marta de León-Contreras
Universidad Autónoma de Madrid

In this talk we shall give Hölder and Schauder estimates for discrete fractional derivatives as well as for the fractional parabolic harmonic oscillator.

We present the discrete fractional derivatives and integrals, and we show some regularity results when the space is a mesh of length h. In this case, see [1], Hölder and Schauder estimates are obtained by means of pointwise estimates. This kind of results have been obtained recently for the fractional discrete Laplacian, see [2].

For the parabolic harmonic oscillator, the estimates are obtained by using semigroup theory. In fact the "adapted spaces" to this operator are defined by

$$\Lambda^{\alpha/2,\alpha}_{t,\mathcal{H}_x} := \left\{ f \in L^\infty(\mathbb{R}^{n+1}) : \left\| \tfrac{\partial^k}{\partial y^k} P_y f \right\|_{L^\infty(\mathbb{R}^{n+1})} \leq C y^{-k+\alpha}, \;\; y > 0, \right\}, \; \alpha > 0.$$

Here $k = [\alpha] + 1$ and P_y is the Poisson semigroup associated with the operator. When the operator is the Laplacian, the parabolic "adapted spaces" $\Lambda^{\delta/2,\delta}_{t,x}$ are defined in [4], where it is shown that they coincide with the parabolic spaces $\mathcal{C}^{\delta/2,\delta}$ introduced by Krylov in [3], when δ is not an integer. In our case, when α is not an integer, the spaces $\Lambda^{\alpha/2,\alpha}_{t,\mathcal{H}_x}$ coincide with a version of parabolic spaces $\mathcal{C}^{\delta/2,\delta}$ defined as in Krylov, but adapted to the harmonic oscillator. In addition, if the function f does not depend on t, the spaces coincide with the Hermite Hölder spaces defined in [5].

Finally we shall show an approximation theorem of the discrete fractional derivatives to the continuous fractional derivatives, for functions in the discrete Hölder spaces. This result also allows us to prove the coincidence, for good enough functions, of the Marchaud and Grünwald-Letnikov fractional derivatives at every point and the speed of convergence to the Grünwald-Letnikov fractional derivative, see [1].

References

[1] L. Abadías, M. De León-Contreras, J. L. Torrea, Non-local fractional derivatives. Discrete and continuous, *J. Math. Anal. Appl.* **449** (2017), 734–755.

[2] Ó. Ciaurri, L. Roncal, P. R. Stinga, J. L. Torrea, J. L. Varona, Nonlocal discrete diffusion equations and the fractional discrete laplacian, regularity and applications, arXiv:1608.08913.

[3] N. V. Krylov, *Lectures on Elliptic and Parabolic Equations in Hölder spaces*, Graduate Studies in Mathematics, American Mathematical Society, 1996.

[4] P. R. Stinga, J. L. Torrea, Regularity theory and extension problem for fractional nonlocal parabolic equations and the master equation, to appear in *SIAM J. Math. Anal.*

[5] P. R. Stinga, J. L. Torrea. Regularity theory for the fractional harmonic oscillator, *J. Funct. Anal* **260** (2011), 3097–3131.

Optimization with respect to order in a fractional diffusion model: analysis and approximation

Abner J. Salgado

The University of Tennessee, Knoxville

We consider an identification problem, where the state u is governed by a fractional elliptic equation and the unknown variable corresponds to the order $s \in (0, 1)$ of the operator. We study the existence of an optimal pair $(\bar{u}, \bar{s})$ and provide sufficient conditions for its uniqueness. We develop semi-discrete and fully discrete algorithms to approximate the solution and provide an analysis of their convergence properties. We present numerical illustrations that confirm and extend our theory. This is joint work with E. Otárola and H. Antil.

Nonlocal mechanics models for anisotropic media

Pablo Seleson

Oak Ridge National Laboratory

Peridynamics is a nonlocal reformulation of classical continuum mechanics, suitable for material failure and damage simulation. Originally, this nonlocal theory was presented as the bond-based peridynamics theory, for which the material response of an isotropic medium is limited by a fixed Poisson's ratio. To overcome this limitation, the state-based peridynamics theory was developed. Applications in peridynamics to date cover a wide range of engineering problems; however, the majority of those applications employ isotropic material models. Only recently, a limited number of anisotropic peridynamic models were developed. In this talk, we will first survey the different classes of anisotropic material models in classical linear elasticity, and we will present a peridynamic framework to represent anisotropic materials. We will then show a classification and a hierarchy of anisotropic peridynamic models, and we will discuss their relation to classical elasticity as well as restrictions arising from a bond-based interaction assumption.

Regularity theory for non local in time operators

Alexis F. Vasseur

The University of Texas at Austin

In this talk, we will present new applications of the De Giorgi method to show the regularity of solutions to nonlocal operators. We will focus on the case of fractional derivatives in time. Those equations are important for the modeling of memory effects as hysteresis. This is a joint work with Mark Allen and Luis Caffarelli.

What are the classical boundary conditions for the fractional Laplace operator?

Mahamadi Warma

University of Puerto Rico (Rio Piedras Campus)

In this talk we characterize all the classical boundary conditions (Dirichlet, Neumann and Robin) associated with the fractional Laplace operator or/and the regional fractional Laplace operator on bounded subsets of $\mathbb{R}^N$. We also give some well-posedness and regularity results of solutions to the associated elliptic and parabolic problems. Finally we introduce a fractional Dirichlet to Neumann operator associated with the regional fractional Laplacian.

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Participants

	Surname, Names	Institution
1	Abedin, Farhan	Temple University
2	Antil, Harbir	George Mason University
3	Bacharach, Max	Iowa State University
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5	Bezerra de Matos, Rodrigo	Michigan State University
6	Bhatnagar, Manas	Iowa State University
7	Biala, Toheeb Ayinde	Middle Tennessee State University
8	Biswas, Animesh	Iowa State University
9	Borthagaray, Juan Pablo	Universidad de Buenos Aires
10	Bouck, Lucas	George Mason University
11	Caffarelli, Luis A.	The University of Texas at Austin
12	Chen, Yanlai	University of Massachusetts Dartmouth
13	Choi, Brian Jongwon	Boston University
14	D'Elia, Marta	Sandia National Laboratories
15	Daws, Joseph	The University of Tennessee, Knoxville
16	De León, Marta	Universidad Autónoma de Madrid, Spain
17	Du, Shukai	University of Delaware
18	Duque, Luis	The University of Texas at Austin
19	Eruslu, Hasan Huseyin	University of Delaware
20	Evans, James W.	Iowa State University
21	Fazly, Mostafa	University of Texas at San Antonio
22	Gillcrist, David Joseph	Missouri University of Science & Technology
23	Glusa, Christian	Sandia National Laboratories
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28	Khaliq, Abdul Q. M.	Middle Tennessee State University
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30	Li, Xingjie Helen	University of North Carolina at Charlotte
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34	Lopez, Juan	University of Houston
35	Luo, Songting	Iowa State University
36	Mamikon, Gulian	Brown University
37	Narayan, Akil	The University of Utah
38	Nguyen, Xuan Hien	Iowa State University

39	Nochetto, Ricardo H.	University of Maryland, College Park
40	Nuguid, Maria Eloisa	San Francisco State University
41	O'Driscoll, Mary	Iowa State University
42	Plackowski, Kenneth	University of Arizona
43	Rodriguez Quinones, Leoncio	Iowa State University
44	Rossmanith, James	Iowa State University
45	Sacks, Paul	Iowa State University
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49	Seleson, Pablo	Oak Ridge National Laboratory
50	Shankar, Ravi	University of Washington
51	Stinga, Pablo Raúl	Iowa State University
52	Tian, Xiaochuan	Columbia University
53	Tong, Jiajun	Courant Institute New York University
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61	Weber, Eric	Iowa State University
62	Weerasinghe, Ananda	Iowa State University
63	Wei, Peng	Texas A&M University
64	Wells, Kelsey	University of Nebraska-Lincoln
65	White, Laura	University of Nebraska-Lincoln
66	Wright, Cory	University of Nebraska-Lincoln
67	Yan, Jue	Iowa State University
68	Yin, Peimeng	Iowa State University
69	Zhang, Yuming	University of California, Los Angeles

Sponsors

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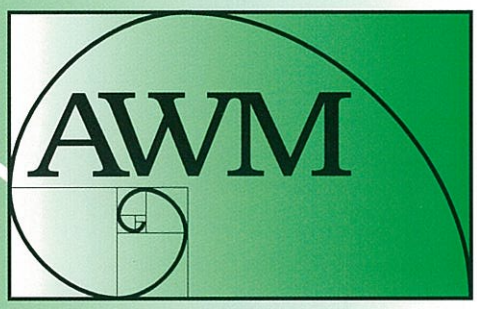

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Mentor Network matches mentors with girls and women interested in a career in mathematics.

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M. Gweneth Humphreys Award to recognize outstanding mentorship.

Essay Contest for K-12 students and undergraduates.

Alice T. Schafer Prize to an undergraduate woman who excels in mathematics.

Ruth I. Michler Memorial Prize to a recently tenured woman for a research semester in the Mathematics Department at Cornell University.

Dissertation Prize for an exceptional dissertation in the mathematical sciences by a woman PhD candidate.

Student Chapter Awards to recognize outstanding achievements in chapter activities among the AWM student chapters.

Research Prizes for early career women

- **AWM-Joan & Joseph Birman Research Prize in Topology and Geometry**
- **AWM-Microsoft Research Prize in Algebra and Number Theory**
- **AWM-Sadosky Research Prize in Analysis**

Travel Grants for women to attend conferences or develop mentoring relationships.

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For further information contact: awm@awm-math.org or (703) 934-0163

NSFE 2017 Schedule

Time	**August 17**	**August 18**	**August 19**
9:00-10:00am		Caffarelli	Nochetto
10:00-10:30am		coffee break	coffee break
10:30-11:00am		Nochetto	Salgado
11:00-11:30am			Warma
11:30-11:45am		short break	
11:45am-12:15pm		Vasseur	
12:15-12:20pm		School photo	
12:20-1:15pm		lunch break	
1:15-1:45pm	Registration	lunch break	
1:45-2:00pm	Opening remarks	lunch break	
2:00-3:00pm	Caffarelli	Caffarelli	
3:00-3:30pm	coffee break	coffee break	
3:30-4:00pm	Nochetto	De León	
4:00-4:30pm		D'Elia	
4:30-4:45pm	short break		
4:45-5:15pm	Seleson		